Optimal Power Flow Problem Solved by Using Distributed Sobol Particle Swarm Optimization

P. Wannakarn[a], S. Khamsawang[b], S. Pothiya[b] and S. Jiriwibhakorn[b]

[a] Electrical Engineering Department, Faculty of Engineering Rajamangala University of Technology Phra Nakhon of Engineering Faculty, Email: v_poonsri555@yahoo.com

[b] Electrical Engineering Department, Faculty of Engineering, King Mongkut's Institute of Technology Ladkrabang, Bangkok, Thailand, 11520, Email: k_suwit999@yahoo.com

***Abstract*—The Distributed Sobol particle swarm optimization (DSPSO) algorithm was studies for solving optimal power flow problem (OPF), in this paper. In the proposed method, swarm size of the particles is separated in multi-groups and searching procedure is divided according with the swarm group. Reducing search space and high cost elimination are concluded in the DSPSO. The DSPSO was tested for solving two sizes of the OPF problem, six bus test system and IEEE-30 bus test system respectively. The numerical results obtain from the DSPSO were compare with many optimization methods, namely bee colony algorithm (BA), differential evolution algorithm (DE), genetic algorithm (GA), particle swarm optimization (PSO) and tabu search algorithm (TSA). The results show that the proposed method had faster convergence and better solution than the rest methods.**

I. INTRODUCTION

The optimal power flow (OPF) problem has been well studied over the past few decades [1–4]. The OPF problem could be treated as a nonlinear optimization problem with nonlinear objective function and subject to several equality and inequality constraints. Many optimization techniques have been applied to solve this problem. Conventional techniques such as Newton method, gradient methods, linear programming, dynamic programming and interior point methods often have problems of convergence and difficulties in locating the global optimum [1-5]. These methods rely on convexity to obtain the global optimum solution and as such are forced to simplify relationships in order to ensure convexity. During the last year, non-conventional methods such as evolutionary programming (EP) [6-7], particle swarm optimization (PSO) [8], tabu search algorithm (TSA) [9], genetic algorithm (GA) [10] and [16] simulated annealing (SA) [11], differential evolution (DE) [14-15] and had been applied to solve the OPF problem.

This paper proposed the distribution procedure of standard particle swarm optimization for solving the optimal power flow. The proposed approach tested on two test system, six bus test system and IEEE 30-bus test system. Simulation results obtained from empirical tests are reported, shown and compared with many non-conventional optimizations appeared in the literatures.

II. PROBLEM FORMULATION

A major difficulty of the OPF problem is the nature of the control variables since some of them are continuous and others are discrete. Optimal values are computed in order to achieve a certain goal subjected to number of equality and inequality constraints. In general, the OPF problem can be presented as:

$$\text{Min } f(x,u) \tag{1}$$

Subjected to

$$g(x,u)=0 \tag{2}$$

$$h(x,u)\le 0 \tag{3}$$

where Min $f(x,u)$ is the objective function. Generally, $g(x,u)=0$ represents the real and reactive power balance equations and $h(x,u)\le 0$ represents security limits. u is a set of control variables, x is a set of dependent variables. The equality constraints (2) are the nonlinear power flow equations which are formulated as follows:

$$0 = P_{Gi} - P_{Di} - V_i \sum_{j=1}^{n} V_j \left(G_{ij}\cos\theta_{ij} + B_{ij}\sin\theta_{ij} \right) \qquad i=1,\ldots,NB \tag{4}$$

$$0 = Q_{Gi} - Q_{Di} - V_i \sum_{j=1}^{n} V_j \left(G_{ij}\sin\theta_{ij} - B_{ij}\cos\theta_{ij} \right) \qquad i=1,\ldots,NB \tag{5}$$

where NB is the total number of system buses, P_{Gi} and Q_{Gi} are the active and reactive power generations of bus i, P_{Di} and Q_{Di} are the active and reactive power loads of bus i; V_i is the voltage magnitude of bus i; θ_{ij} is the voltage angle difference between bus i and j; G_{ij} and B_{ij} are transfer admittance between bus i and j.

The inequality constraints (3) are the system operating constraints, including

(a) Generator constraints:

$$V_{G_i}^{\min} \le V_{G_i} \le V_{G_i}^{\max}, \qquad i \in NG \tag{6}$$

$$P_{G_i}^{\min} \le P_{G_i} \le P_{G_i}^{\max}, \qquad i \in NG \tag{7}$$

$$Q_{G_i}^{\min} \le Q_{G_i} \le Q_{G_i}^{\max}, \qquad i \in NG \tag{8}$$

where NG is the number of generator buses.

(c) Security constraints: For secure operation, the transmission line loading S_{l_i} is restricted by its upper limits $S_{l_i}^{\max}$ as:

$$V_{L_i}^{\min} \le V_{L_i} \le V_{L_i}^{\max}, \qquad i \in NB \tag{9}$$

$$S_{l_i} \le S_{l_i}^{\max}, \qquad i \in nl \tag{10}$$

where nl is the number of transmission lines.

The most common method for solving nonlinear constrained optimization problems is to transfer a constrained optimization problem into an unconstrained [6-9]. Thus, the objective function (1) is generalized as follows:

$$f(x,u) = \sum_{i=1}^{NG} F_i(P_{Gi}) + \lambda_P (P_{G_1} - P_{G_1}^{\lim})^2 + \lambda_V \sum_{i=1}^{NL} (V_{L_i} - V_{L_i}^{\lim})^2 + \lambda_Q \sum_{i=1}^{NG} (Q_{G_i} - Q_{G_i}^{\lim})^2 + \lambda_S \sum_{i=1}^{nl} (S_{l_i} - S_{l_i}^{\max})^2 \tag{11}$$

where

$$F_i(P_{Gi}) = a_i + b_i P_i + c_i P_i^2 \tag{12}$$

where a_i, b_i, c_i are cost coefficients of generating unit i, $\lambda_P, \lambda_V, \lambda_Q$ and λ_S are penalty factors

III. DSPSO BASED OPTIMAL POWER FLOW

A. Particle swarm optimization

Kenedy and Eberhart first introduced PSO in 1995 [21]. The mechanism of this algorithm is the searching parallel processing, which uses a group of individuals similar to other artificial intelligences (AI) as heuristic optimization techniques [22-23]. In the PSO process, changing the position of each population is flying around in a multidimensional search space.

During search, each particle changes its position by own information of its previous generation and use of best position of neighboring particles. Therefore, the particle change based on the set of particle, particles neighboring and its information history (called velocity of particles).

Let x and v are particle position and velocity of particles in the search space. Therefore, the position of i^{th} particle represented as vector $x_i = [x_{i1}, x_{i2}, ... x_{id}]$ and the velocity of i^{th} particle represented as vector $v_i = (v_{i1}, v_{i2}, ... v_{id})$ in the $d-$dimensional space. The best position of particle i^{th} is stored and represented as $pbest_i = (pbest_{i1}, pbest_{i2}, ... pbest_{id})$. The best position among all particles is represented as $gbest$. Using the previous information and own experienced, the updated velocity of particle i^{th} is calculated by the following equation.

$$v_i^{(t+1)} = \omega \times v_i^{(t)} + c_1 \times r_1 \times (pbest_i - x_i^{(t)}) + c_2 \times r_2 \times (gbest - x_i^{(t)}) \tag{13}$$

$$\omega = \omega_{\max} - \frac{\omega_{\max} - \omega_{\min}}{iter_{\max}} \times iter \tag{14}$$

$$x_i^{(t+1)} = x_i^{(t)} + v_i^{(t+1)}, \ \mathrm{i} = 1, 2, ..., \mathrm{n}, \tag{15}$$

where $v_i^{(t)}$ is velocity of particle i at generation t, which $V_d^{\min} \le v_i^{(t)} \le V_d^{\max}$, c_1 and c_2 are weighting factors, r_1 and r_2 is random number between 0 and 1, t is number of generations (or iterations), ω is inertia weight factor, $\omega_{\min}$ and $\omega_{\max}$ are initial and final inertia weight factor, $iter$ is current iteration number, $iter_{\max}$ is maximum iteration number and $x_i^{(t)}$ is position of particle i^{th} at generation t.

B. Distributed Sobol particle swarm optimization (DSPSO)

The velocity mechanism of the conventional PSO is calculated by using equation (13 - 14), and then the position of particle is modified by using equation (15).

This paper introduces the new PSO process which splits the group of particles to many subgroups. Each subgroup has a memory which stores its current particle, the best particle and the best particle of among all the particles. All subgroups will use these memories together in order to look into the best particle among all groups ($Gbest$). The cognitive socials are modeled by the best particle among all the particles in the group and the best particle among all the particles in all groups. The technique distributes the PSO to many groups with Sobol inertia factor is called Sobol distributed particle swarm optimization (DSPSO). The velocities and position updating of all the groups are executed by using equation (16) and (17), respectively.

$$V_i^{(t+1)} = K \times \begin{pmatrix} \omega_{sb} \times V_i^{(t)} + c_1 \times r_1 \times (pbest_i - x_i^{(t)}) \\ + c_2 \times r_2 \times (gbest - x_i^{(t)}) \\ + c_4 \times r_4 \times (Gbest - x_i^{(t)}) \end{pmatrix} \tag{16}$$

$$V_i^{(t+1)} = K \times \begin{pmatrix} \omega_{sb} \times V_i^{(t)} + c_1 \times r_1 \times (pbest_i - x_i^{(t)}) \\ + c_2 \times r_2 \times (gbest - x_i^{(t)}) \\ + c_3 \times r_3 \times (pbest - x_{ij}^{(t)}) \\ + c_4 \times r_4 \times (Gbest - x_i^{(t)}) \end{pmatrix} \tag{17}$$

$$K = \left| \frac{2}{2 - c - \sqrt{c^2 - 4 \times c}} \right|, \quad c = c_1 + c_2 \text{ and } c > 4 \tag{18}$$

where K is constriction factor, ω_{sb} is the Sobol inertia factor, $Gbest$ is the best particles among all groups, r_3 and r_4 is random number between 0 and 1, and c_3 and c_4 is the accelerator factor. Figure 1 shows the flow chart of applying the proposed method for solving this problem

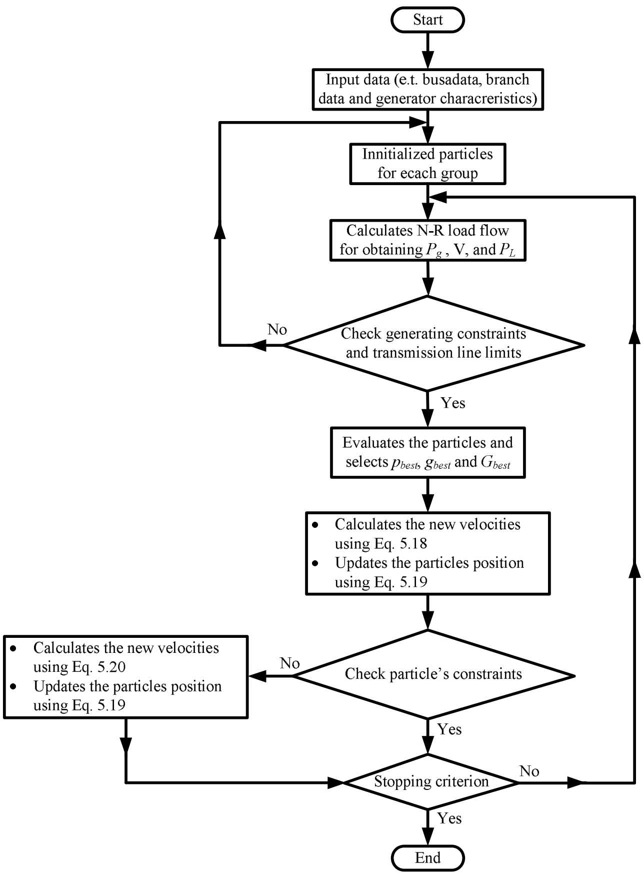

Figure 1 Flow chart of DSPSO applied to the OPF problem

IV. SIMULATION RESULTS

The DSPSO, BA [13], DE [14-15], GA [16], PSO [17-19] and TSA [20-22] methods are implemented in MATLAB language and executed on an Intel Core 2 Duo 3.0 GHz personal computer with a 4.0 GB of RAM. In order to illustrate the effectiveness and robustness of the proposed method, two test systems have been examined and considered. The first is the 6-bus test system [5] and the second one is the IEEE 30-bus test system [6-12]. The effectiveness of the proposed method is compared with several methods.

A. Six bus system

Figure 2 shows the single-line diagram of six-bus test system with 4 generating units, 7 transmission lines and total system load demand is 600 MW. Bus data and line data of this test system are shown in Ref. [5]. The objective function of this case study is quadratic function like as equation (12).

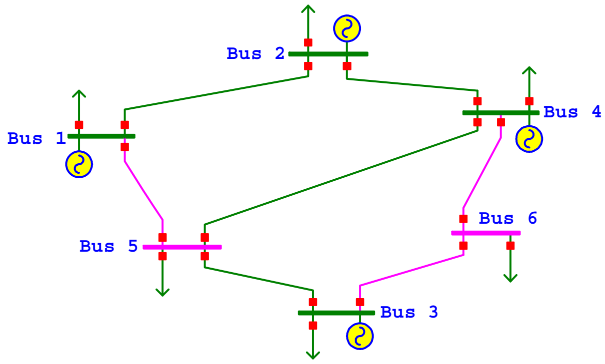

Figure 2 Schematic diagram of 6-bus test system

For applying the proposed method, a particle size of each group is 5 and maximum iteration is taken as 100. Simulation results obtained from DSPSO compared with BA, DE, GA, PSO and TSA and shown in Table 1 and Table 2, respectively. Table 1 shows the results of proposed method and other methods after performing 100 independent runs, which include the minimum, average, maximum generation cost, computational time and standard deviation. It can be observed that the results obtained from DSPSO had outperformed than the rest methods in term of average generation cost, average computational time and standard deviation of solutions.

TABLE 1
SIMULATION RESULTS OF 6-BUS TEST SYSTEM

Methods	Generation cost ($/h) Min	Mean	Max	CPU times (s)	Standard deviation
BA	7823.535	7823.624	7823.699	3.18	0.04
DE	7823.537	7823.631	7823.698	1.42	0.04
GA	7823.539	7823.638	7823.7	1.7	0.04
PSO	7823.537	7823.626	7823.697	1.23	0.04
TSA	7823.551	7823.626	7823.699	1.71	0.04
DSPSO	***7823.535***	***7823.614***	***7823.698***	***0.5***	***0.04***

The best solution is founded by each method are slightly different and reported in Table 2. Figure 3 illustrates the convergence characteristic of the proposed method compared with five methods, seen that the DSPSO has converge to near global optimum faster than BA, DE, GA, PSO and TSA.

TABLE 2
BEST RESULTS COMPARISON OF THE DSPSO-TSA WITH FIVE METHODS

Power output (MW)	BA	DE	GA	PSO	TSA	DSPSO
P_1	91.20	91.03	91.29	91.25	90.65	91.02
P_2	196.35	196.37	196.72	196.50	196.12	196.64
P_3	86.48	86.24	86.20	86.10	87.10	85.47
P_4	236.79	237.21	236.64	237.00	236.89	237.80
PG (MW)	610.81	610.84	610.85	610.86	610.76	610.93
PL (MW)	10.81	10.84	10.85	10.86	10.76	10.84
TC ($/h)	7823.54	7823.54	7823.54	7823.54	7823.55	***7823.54***

* PG is total real power generation of system.
* PL is total real power loss.
* TC is total generation cost

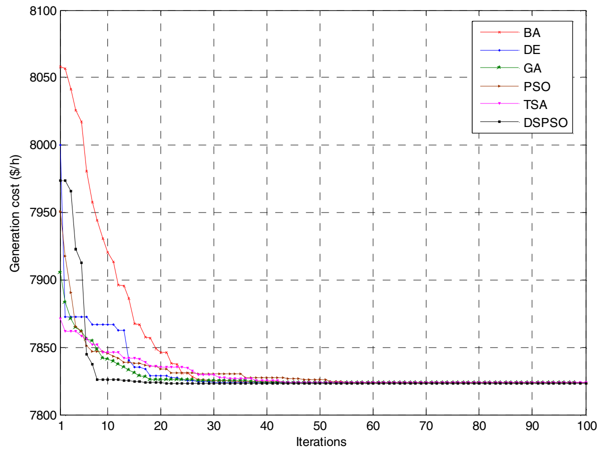

Figure 3 Convergence characteristics of six methods

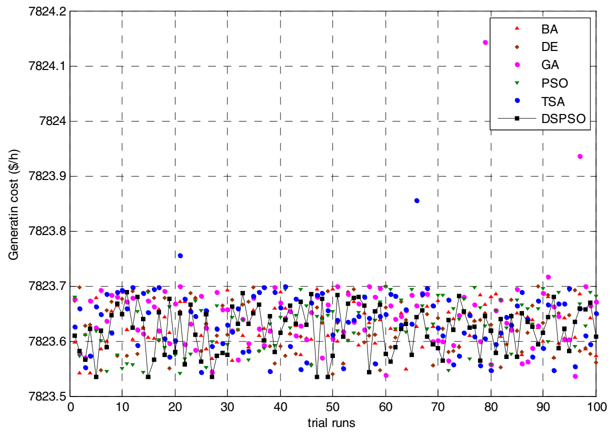

Figure 4 Comparison of solution profiles of the 6-bus OPF problem

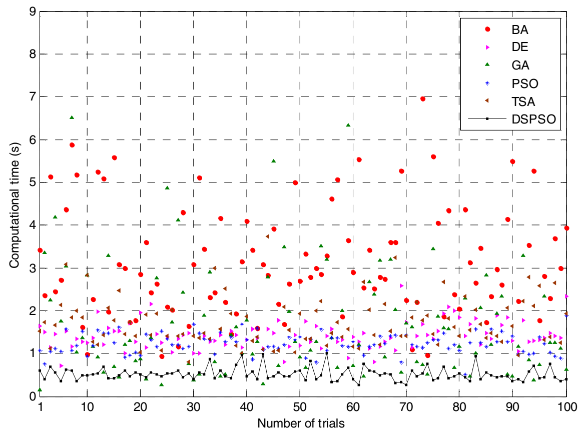

Figure 5 Computational time comparison in 100 trials

The solution profiles of 100 different trial runs of the proposed method compared with the rest methods shown in Figure 4. Evident that the results of other methods had probably obtained the local optimum, thus the standard deviation value of generation cost of the proposed method has lower than other methods. Figure 5 illustrates the computational profiles of the DSPSO compared to other optimization methods. Figure 6 demonstrates the example and some value of c_3 and c_4. The figure shows that, c_4 is fixed while c_3 varied from 1 to 2. The best value of these parameter obtained from empirically tests are $c_3 = 1.40$ and $c_4 = 0.85$.

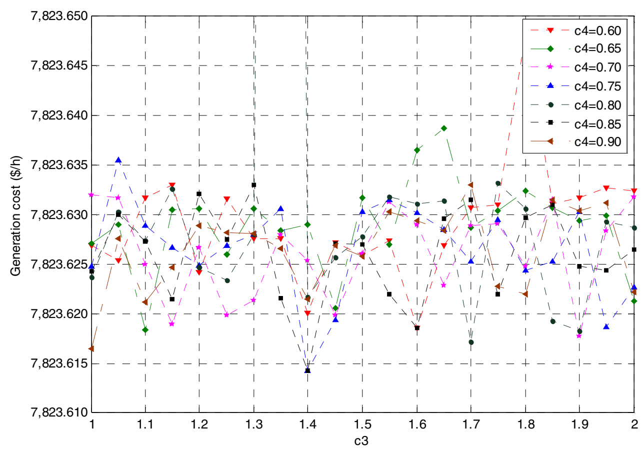

Figure 6 Solutions empirically obtained from testing c_3 and c_4

B. IEEE 30-bus test system

In this section performance of proposed approach for solving the optimal power flow is tested on IEEE 30 bus system. The system consist of six generating units committed at bus 1, 2, 5, 8, 11 and 13, forty-one transmission lines, four tap-changing transformers installed in lines 4-12, 6-9, 6-10 and 27-28 and two capacitors installed at bus five and twenty-four [6-12].

Table 3 shows the robustness of the proposed methodology is carried out for 100 trial runs with difference initial starting points (initial populations or particles), it can be observed that the simulation results of the proposed method had better than other method namely, BA, DE, GA, PSO and TSA in term of minimum generation cost, average generation cost, average executed time and standard deviation. Table 4 depicts the best generation cost of several methods which compared with DSPSO-TSA. The results show that the proposed method gives minimum solution than the rest methods.

TABLE 3
COMPARISON OF RESULTS OBTAINED FROM SIX METHODS

	Generation cost ($/h)			CPU	Standard
Methods	Min.	Mean	Max.	times (s)	deviation
BA	801.8484	801.8696	801.8770	10.15	0.006
DE	801.8490	801.8633	801.8699	11.44	0.005
GA	801.8539	801.8727	801.9050	17.88	0.006
PSO	801.8701	801.8861	802.1843	8.13	0.035
TSA	801.8517	801.8633	801.8700	24.78	0.005
DSPSO	***801.8462***	***801.8616***	***801.8699***	***5.68***	***0.005***

TABLE 4
BEST RESULT OF SIX METHODS OBTAINED FROM SOLVING OPF PROBLEM

Power output (MW)	BA	DE	GA	PSO	TSA	DSPSO
P_1	177.138	177.03	176.08	176.68	177.24	177.59
P_2	48.96	48.82	50.01	48.35	48.91	48.79
P_3	21.40	21.25	21.49	21.79	21.65	21.33
P^5	21.19	21.47	21.66	21.50	21.49	21.78
P_8	12.22	12.36	12.23	12.17	11.66	12.08
P_{13}	12.04	12.01	11.31	12.31	12.01	12.024
PG (MW)	292.95	292.95	292.76	292.79	292.96	293.59
PL (MW)	9.632	9.621	9.364	9.565	9.632	9.668
TC ($/h)	801.848	801.849	801.854	801.870	801.852	***801.846***

V. CONCLUSIONS

The standard PSO reported by Kenedy and. Eberhart in 1995 and 1998. The distributed the process of the standard PSO, the new velocity equations and application of the Sobol sequences are proposed techniques to improve the solution quality and convergence time of standard PSO and called distributed Sobol particle swarm optimization (DSPSO).

The effectiveness of the proposed method is tested with the two test systems, six bus system and IEEE 30-bus system, respectively. The simulation solutions are investigated with 100 difference trials and compared with several optimization methods. Obviously, the proposed approach can improve the performance of standard PSO and yield the best solution qualities than BA, DE, GA, PSO and TSA depicted in term of the best solution, the average solution, the computational time and the standard deviation of solution shown in Table 1-Table 4. In addition, the DSPSO can converge to the global optimum with shorter average executed time than the rest methods as illustrated in Figure 3, Table 1 and Table 3.

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